Visualization and Analysis of Trade-offs in Many-Objective Optimization: A Case Study on the Interior Permanent Magnet Motor Design

Rodrigo Silva[1], Armin Salimi[1], Min Li[1], Alan R. R. Freitas[2], Frederico G. Guimarães[3], and David A. Lowther [1]

[1]Department of Electrical and Computer Engineering, McGill University, Montreal, QC, Canada
[2]Department of Computer Science, Universidade Federal de Ouro Preto (UFOP), Ouro Preto, MG, Brazil
[3]Department of Electrical Engineering, Universidade Federal de Minas Gerais (UFMG), Belo Horizonte, MG, Brazil

Presentation and visualization of trade-off solutions in many-objective optimization problems are difficult due to the large number of solutions in a hyper dimensional objective space. A recently proposed tool, known as aggregation tree (AT), can be used to analyze the degree of conflict between groups of objectives in a many-objective problem. In this paper, we present a case study on the internal permanent magnet motor design with seven objectives. The results show that the AT is able to provide insight into the electrical machine design problem (in accordance with the common knowledge of physics) as well as guidance in the reduction of objectives.

Index Terms—**Design Optimization, Multi-objective Optimization, Objective Reduction, Permanent Magnet Motors.**

I. Introduction

THE DESIGN of electrical machines are often formulated as optimization problems in which the objective functions are defined over key performance features. The most commonly used objectives in the literature of electric motor design are size or weight of the machine [1], [2], material cost [3], maximum or average torque [4], [5], torque ripple [3]–[5], and efficiency or losses (core and copper) [1], [2], [4]. In general, some of these objectives are in conflict, what requires the solution of a multi-objective optimization problem (MOOP). Unfortunately, the difficulty of the MOOPs increases with the number of objectives, and the solution of the many-objective problem (with four or more objectives) becomes problematic for standard evolutionary multi-objective (EMO) algorithms [6]. Besides, even if a reasonable set of solutions can be found, it is hard to present and visualize them in the high-dimensional objective space. In this context, we present a methodology, known as Aggregation Tree (AT) [7], which can be used by designers and practitioners to understand the relationships between different objectives and possibly aid them to reduce the problem size in an effective way. In this paper, the visualization of the objectives through the AT is demonstrated using the design of an Interior Permanent Magnet (IPM) machine as an example. The AT provides insight into the IPM design problem as well as guidance in the reduction of objectives. In this way, the designer has empirical knowledge to guide him in the formulation and solution of the optimization problem.

II. Many-Objective Optimization

A multi-objective optimization problem can be formulated as:

$$minimize\ \ (f_1(\mathbf{x}), f_2(\mathbf{x}), ..., f_k(\mathbf{x})),\ \ \mathbf{x} \in \mathcal{F} \tag{1}$$

where $f_i(\cdot)$ is the i-th objective function to be minimized and $\mathcal{F}$ represents the feasible set. Solving the problem defined in (1) involves finding a set of Pareto-optimal solutions, i.e. solutions that cannot be improved in one objective without compromising their performance in another one.

The most commonly used methods for multi-objective problems are the evolutionary algorithms with Pareto-based fitness assignment. Despite of their success, if the number of objectives is greater than four, Pareto-ranking does not work as a good discriminator of solutions as most of them are non-dominated. In addition, the number of solutions that are required to approximate the Pareto front increases exponentially with the number of objectives, and their visualization in the objective space becomes difficult. Due to these additional difficulties this kind of problem has been considered by the related literature as a special class of multi-objective problems, referred to as Many-objective problems [6].

III. Aggregation Tree

Aggregation Trees [7] can be used to easily visualize the relationships between objectives of a Many-objective problem, group objectives according to their reducibility, and show quantitatively the amount of conflict between them.

To build the tree, at each iteration the algorithm finds the least conflicting pair of objectives and aggregates them in a compound objective. Thus, at each iteration the algorithm reduces the number of objectives by one and as it proceeds the new compound objective may be further combined with the remaining ones. The conflict C_{ab} between two objectives f_a and f_b is given by Equation (2). Basically, the most usual measure of conflict is a normalized sum of rank differences.

$$C'_{ab} = \sum_{i=1}^{i=n} |\mathbf{R}_{ia} - \mathbf{R}_{ib}| \tag{2a}$$

$$c_{max} = \sum_{i=1}^{i=n} |2i - n - 1| \tag{2b}$$

$$C_{ab} = \frac{C'_{ab}}{c_{max}} \tag{2c}$$

Given a set of solutions $\mathbf{X} = \{\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_n\}$, $\mathbf{R}_{ij}$ is the rank of solution $\mathbf{x}_i$ in the objective j and c_{max} is the maximum possible conflict in a set of n solutions.

IV. IPM Design

In this paper, we explore the Aggregation Tree to understand and to reduce the number of objectives in the many-objective design optimization of an IPM motor. The design variables, depicted in Fig. 1, are defined as:

x_1: Winding slot depth (mm), $0.5 \le x_1 \le 18$
x_2: Winding slot length (mm), $4 \le x_2 \le 21.5$
x_3: Winding slot angle (deg), $1 \le x_3 \le 14$
x_4: Depth of PM (mm), $6 \le x_4 \le 26$
x_5: Thickness of PM (mm), $0.5 \le x_5 \le 20$
x_6: Width of PM (mm), $0.5 \le x_6 \le 39$
x_7: Lower width of the rotor slot(mm), $0.5 \le x_7 \le 20$
x_8: Upper width of the rotor slot(mm), $0.5 \le x_8 \le 39$

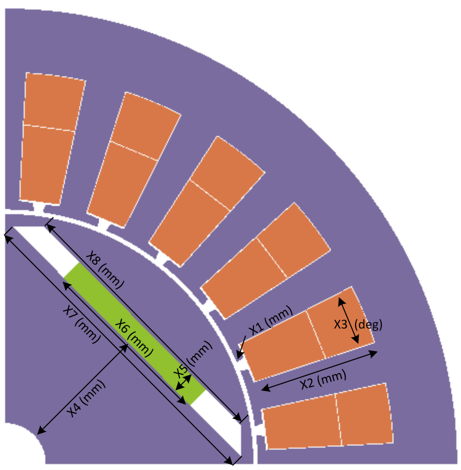

Fig. 1. IPM Design Problem

Based on a review of several literature papers related to electrical machine design [1]–[5], the following seven objectives were selected for the problem formulation:

f_1: Average Torque
f_2: Torque Ripple
f_3: Permanent Magnet Volume
f_4: Rotor Losses
f_5: Stator Losses
f_6: Starting Torque
f_7: Ohmic Losses

V. Results

With the defined problem, a Latin Hypercube was utilized to sample the design space in 10000 different locations. These samples were then used to generate the aggregation tree depicted in Figure 2. Parent nodes represent possible aggregations and the conflict between the children nodes. As it can be seen, the aggregation tree confirms usual assumptions such as the low conflict between average torque (f_1) and starting torque (f_6); and low conflict between stator losses (f_5) and ohmic losses (f_7).

Several scenarios can be derived from the AT for the reformulation of the design problem. For instance, we can select just one objective from low conflict branches (e.g., f_1,

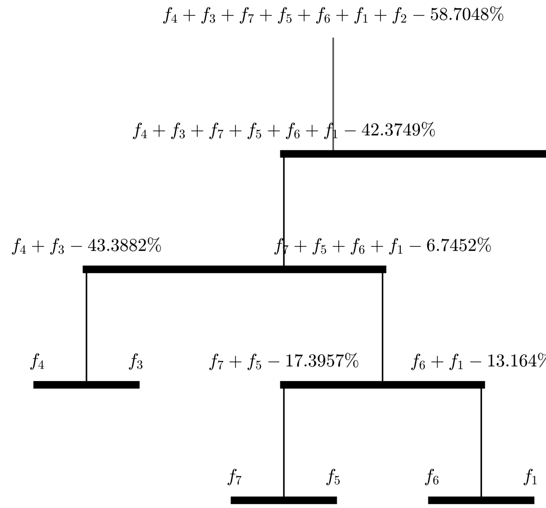

Fig. 2. Aggregation Tree

f_2, f_3 and f_5) for the new problem or aggregate the objectives in the neighboring nodes to reduce number of objectives (e.g. $(f_1 + f_6)$, $(f_7 + f_5)$, f_2 ,$(f_3 + f_4)$).

VI. Conclusion

In this paper, we have shown how the designer can enhance his understanding of a many-objective problem through the use of the aggregation tree. Its application to the IPM design has verified some usual assumptions and provided directions for an informed choice of objectives in the design problem. In the full paper, the authors intend to apply optimization to some scenarios suggested by the AT in order to show the actual gain the approach can have in the overall optimization performance.

References

[1] C. Dong-Hyeok, J. Hyun-Kyo, and S. Dong-Joon, “Multiobjective optimal design of interior permanent magnet synchronous motors considering improved core loss formula,” *Energy Conversion, IEEE Transactions on*, vol. 14, no. 4, pp. 1347–1352, 1999.

[2] G. Pellegrino and F. Cupertino, “FEA-based multi-objective optimization of IPM motor design including rotor losses,” in *Energy Conversion Congress and Exposition (ECCE), 2010 IEEE*, pp. 3659–3666.

[3] T. Ohnishi and N. Takahashi, “Optimal design of efficient IPM motor using finite element method,” *Magnetics, IEEE Transactions on*, vol. 36, no. 5, pp. 3537–3539, 2000.

[4] X. Jannot, J. C. Vannier, C. Marchand, M. Gabsi, J. Saint-Michel, and D. Sadarnac, “Multiphysic modeling of a high-speed interior permanent-magnet synchronous machine for a multiobjective optimal design,” *Energy Conversion, IEEE Transactions on*, vol. 26, no. 2, pp. 457–467, 2011.

[5] K. Sung-Il, L. Ji-Young, H. Jung-Pyo, and K. Young-Kyoun, “Application of response surface methodology combined with experimental design for improving torque performance of interior permanent magnet synchronous motor,” in *Electric Machines and Drives, 2005 IEEE International Conference on*, pp. 1665–1668.

[6] H. Ishibuchi, N. Tsukamoto, and Y. Nojima, “Evolutionary many-objective optimization: A short review,” in *Evolutionary Computation, 2008. CEC 2008. (IEEE World Congress on Computational Intelligence). IEEE Congress on*, pp. 2419–2426.

[7] A. R. de Freitas, P. J. Fleming, and F. G. Guimares, “Aggregation trees for visualization and dimension reduction in many-objective optimization,” *Information Sciences*, vol. 298, no. 0, pp. 288 – 314, 2015. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S0020025514011347